

Structuring Light for Investigating Optical Vortices

Andrew Voitiv¹, Mark Siemens²¹Student Contributor, University of Denver²Advisor, Department of Physics, University of Denver

Abstract

Vortices are well known in our world: tornadoes, hurricanes, and quickly stirred iced tea all demonstrate the vortex phenomenon. In addition to these classical fluids, vortices exist in laser light. While classical fluid vortex dynamics is one of the oldest studied physics problems, the study of optical vortices is only a few decades old. Paralleling the community's curiosity of quantized vortices in quantum fluids, such as super fluid helium and Bose-Einstein condensate, there is immense interest in the study of optical vortices. In this article, we cover the basic theory of structuring light to generate optical vortices and then discuss experiments performed to study their characteristics and dynamics.

Keywords: physics – lasers – vortex – optics – holography – vortices

1 STRUCTURED LIGHT: FROM SUNGLASSES, TO TWISTED BEAMS, TO OPTICAL QUANTUM FLUIDS

Vortices are well known in our world: tornadoes, hurricanes, and quickly stirred iced tea all demonstrate the vortex phenomenon. In addition to these classical fluids, vortices exist in laser light. While classical fluid vortex dynamics is one of the oldest studied physics problems, the study of optical vortices is only a few decades old. Paralleling the community's curiosity of quantized vortices in quantum fluids, such as superfluid helium and Bose-Einstein condensate, there is immense interest in the study of optical vortices. This brief article covers the basic theory of structuring light to generate optical vortices and the scientific interest in studying them. I present a brief review of how the Siemens Lab generates and measures these objects.

Structuring light is the process of manipulating streams of photons into desired patterns of amplitude, phase, and polarization. Examples of unstructured light would be the light from the Sun or from iridescent light bulbs; these black body radiators emit all frequencies of visible light (hence the whiteness); it is well known that the light emitted is unpolarized, and intuitively, it is clear that the amplitude and phase are not pre-configured either. Perhaps the most well-known structured light is the light that passes through a pair of polaroid sunglasses. Light, and in general electromagnetic waves, is the result of oscillating perpendicular electric and magnetic fields, generated by vibrating electric charges. In black bodies, these vibrations are random and so the

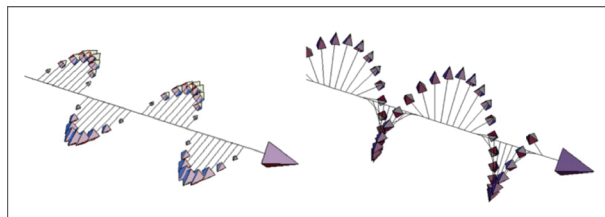


Figure 1. The arrows represent the electric field of a propagating beam. Light with no SAM is linearly polarized (left) and light with SAM is circularly polarized (right). Figure from¹.

subsequent electromagnetic wave varies randomly as it propagates. Polarizers give result to electromagnetic waves with fields that vary with time in a preferred direction, whether it be circularly or linearly, and they can do so by several methods involving transmission, reflection, and refraction.

Photons possess energy $\hbar\omega$ and travel with linear momentum $\hbar\mathbf{k}$, for angular frequency ω and wave-number $k = 2\pi$. Photons are bosons with spin quantum number 1; this means that photons have an intrinsic spin angular momentum \hbar that is either aligned or anti-aligned to the direction of propagation. Being circularly polarized, photons are said to carry a spin angular momentum (SAM) of $\pm\hbar$.

The other “flavor” of angular momentum is orbital angular momentum (OAM). While the term spin is merely a metaphor to label that intrinsic property of angular momentum (the photon, or any other particle, does not actually spin like a top), OAM is a quantum

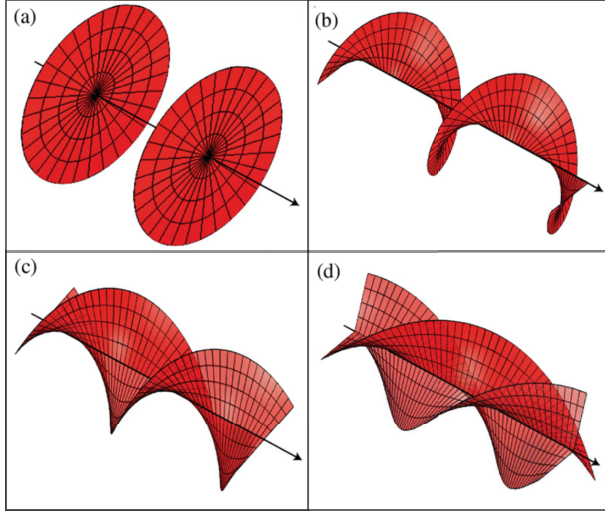


Figure 2. Depiction of the phase fronts of beams of varying OAM values. (a) Is a plane wave: $l = 0$, so there is no azimuthal dependence, and hence a flat phase front. (b), (c), and (d) have OAM values of $l = 1, 2,$ and $3,$ respectively. The l corresponds to the number of twisted turns: the number of 2π phase wraps. Figure from¹.

mechanical operator that is analogous to its classical counterpart. That is, a photon with linear momentum p moving at position r about a fixed reference axis, has orbital angular momentum of $L = r \times p$. The associated quantum number for L_z is l , known as the azimuthal quantum number. Just as photons carry SAM of $\sigma\hbar$ for $\sigma = 1$ in the direction of propagation, photons have OAM of $l\hbar$ with respect to a defined axis (usually the mode center) for any integer l .

Lasers are an example of highly controlled light. They are composed of streams of photons of defined (or even tunable) frequency, the beams can be tightly focused and precisely pointed, the photons are of coherent phase, and the beams can be manipulated with relative ease. If the photons carry spin $\sigma\hbar$, then integrating over the entire beam, the beam is circularly polarized with SAM of $-\hbar$ per photon, with the handedness determined by the sign². Figure 1 depicts the beam polarization. In 1992, Allen et al. demonstrated that laser beams can also carry OAM if the beam has azimuthal phase dependence (azimuthal refers to the angle in the xy -plane, transverse to the axis of propagation, z), with each photon having OAM of $l\hbar$ ³. This azimuthal phase dependence is structured in the function $e^{il\phi}$ and is seen in laser beams with helical wave fronts, as pictured in Figure 2. The helicity arises from the definition of angular momentum along the z -direction (which is the axis of propagation), which requires components of linear momentum in the xy -transverse-plane:

$$L_z = (r \times p)_z = xp_y - yp_x \quad (1)$$

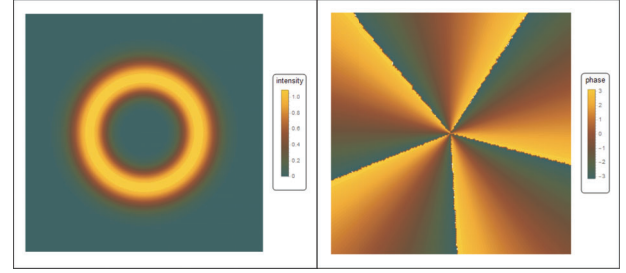


Figure 3. Analytical model of a Laguerre-Gaussian (LG) mode with $l = 5$; the left side depicts the transverse profile intensity and the right side shows the helical twisting, with a total of five phase wraps.

For plane waves, with just the structure e^{ikr} , there are no transverse components of the electromagnetic fields and hence the phase fronts are flat. Even circularly polarized light, for which the field appears to be spinning about the z -axis, has a flat wave front because the field is solely perpendicular to that axis.

Any photon with a helical phase ($e^{il\phi}$) carries OAM. In the discussion of structured light, we look for equations that can describe OAM modes that exist in laser beams. A common set of modes used to model beams with OAM is the Laguerre-Gaussian (LG) modal set. These modes with OAM have the nickname of “donut beams” because their helical phase front results in a “hole”, a phase singularity, in the center and a circularly structured transverse amplitude profile. An example for an LG beam with $l = 5$ is shown in Figure 3. The phase singularity, also known as an optical vortex, is a key characteristic of beams structured with helical phase. This “spot”, as it appears in the donut, is really a line of darkness that runs down the center of the beam along the propagation direction. The vortices are assigned a topological charge corresponding to the OAM value used in structuring that specific topology. For the example below, the vortex has a topological charge of $+5$.

There is great scientific and engineering interest in OAM for its applications in light-matter interactions and for its potential in improving communications technology. For instance, OAM has been shown to act like an “optical spanner” in optical tweezer arrangements, where a beam with OAM can rotate a trapped particle by transferring the OAM⁴. Along those same lines, in the Siemens Lab, there were measurements on the effects of transferring OAM from laser light to electrons in a copper conductor⁵. And in communications, there have been investigations into drastically increasing the bandwidth of data systems by using multiple OAM states in optical fibers^{6;7}.

The optical vortices present in donut beams are not a specialty restricted to just those structures. The interference of two plane waves, say from two equally intense laser beams, results in the usual interference pattern known as a diffraction grating (see the left side of Figure

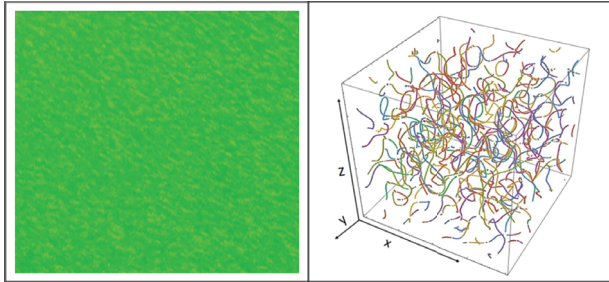


Figure 4. Left side: laser speckle from the interference of a green laser with a green, inflated balloon. The spots of darkness are the optical vortices. Right side: numerically simulated tracks of phase singularities in the propagation in the z -axis of the wave. Figure from ⁸

5). If three or more such plane waves interfere, phase singularities will be generated from the complete destructive interference at points in the field⁹. An example of this is laser speckle, which is the grainy intensity pattern seen when blown-up coherent light (such as a laser beam) reflects from a rough surface; the left side of Figure 4 depicts speckle for a laser beam transmitted through a balloon. Each point of darkness is a phase singularity from the interference of countless plane waves, caused by diffuse reflection from the rough surface. The vortices in this random wave have an intricate structure. As seen in the right side of Figure 4, the phase singularities in random waves form complicated tangles as the wave propagates (as it evolves in time⁸); these topological features are loops and knots of lines of darkness^{10;11}.

In the Siemens Lab, it was shown numerically and experimentally that nonlinear, room-temperature laser light demonstrates quantum turbulent structure in the form of these random waves⁸. That is, by tracking the velocities of vortices as they entwine through the topological features of Figure 4, it was shown that the distribution of vortex velocities matches the characteristic heavy-tailed vortex velocity distributions of atomic Bose-Einstein condensate and superfluid helium^{12;13}. These findings are exciting and disruptive because they demonstrate that classical optics and quantum fluid dynamics are connected by some underlying universal physics—which is not yet fully understood. As the experimental study of typical quantum fluids requires ultra-cold temperatures and complex apparatuses, the accessibility of regular laser light has the potential to expedite and expand the knowledge base of quantum fluid and turbulence dynamics—turbulence being a field which is still short of a complete theory, either for classical or quantum turbulence. Our group's observation of these phenomena led us to coin a new medium: the topological fluid of light (TFL).

We are interested in structuring light, and the observations above were made for random waves. This has propelled the Siemens Lab into controlling how TFL

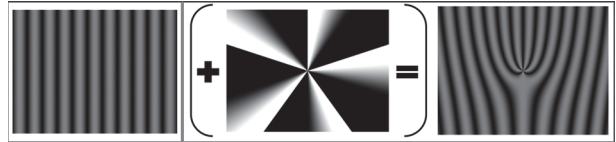


Figure 5. Left side: numerically generated matrix plot of two interfering plane waves, which results in a diffraction grating that can be used to holographically diffract a normal, $l = 0$ beam. Right side: Similar situation, except now one of the interfering plane waves used to construct the grating is imposed with a spiral phase of $e^{il\phi}$ for $l = 5$ (center); this creates a forked diffraction grating (right), which will holographically generate a Hypergeometric-Gaussian (HyGG) mode with $l = 5$ in the first diffracted order.

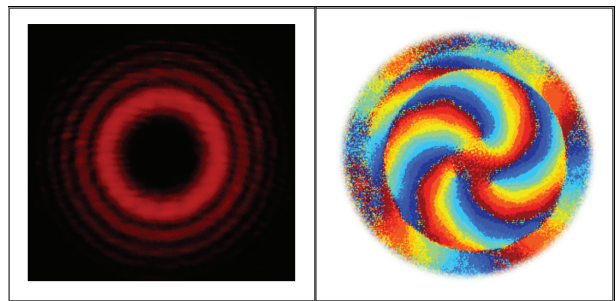


Figure 6. Left side: data of an $l = 4$ Hypergeometric-Gaussian (HyGG) mode generated with a forked diffraction grating similar to that as depicted in Figure 5. Note the presence of multiple rings, which is characteristic of HyGG modes. Right side: the phase data of this image, indicating the four phase wraps around the central phase singularity. This data was obtained by our method of collinear phase shifting digital holography¹⁴

is generated, allowing us to design TFL systems from simple two-vortex systems that can lead to vortex annihilation, to complex vortex arrays that demonstrate steady-state dynamics of braiding. This current work is predicted to establish vortices as quasiparticles analogous to anyons (particles neither being fermions or bosons, restricted to two-dimensional systems); to make a quantum field theory formalism for the interaction of these quasi-particles; and to uncover potential applications for quantum computation. Following, I show methods of generating light with OAM, make reference to our lab's novel method of measuring these modes, and provide preliminary data in this new research on TFL.

2 GENERATION AND MEASUREMENT OF OPTICAL VORTICES

Laser light can be imparted with orbital angular momentum (OAM) in a variety of ways. The simplest method of transforming Gaussian beams into beams with OAM is to use computer generated holograms¹⁵. In using this technique, the hologram is a desired diffraction grating

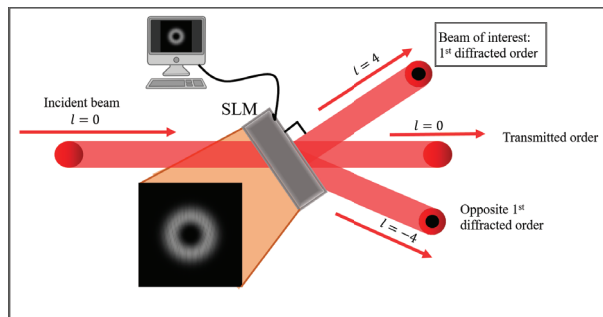


Figure 7. The grating to make a Laguerre-Gaussian (LG) mode of $l = 4$ and $p = 0$ is displayed on the SLM controlled by a computer, and the first diffracted order contains the desired mode. The transmitted order and opposite first diffracted order (as well as all other diffracted orders not depicted) are blocked.

(also known as an interference pattern). In the simplest case, we can make a plain diffraction grating by interfering, numerically, two plane waves and creating a matrix plot of the tabulated interference values, as shown in the left side of Figure 5.

If we wish to generate a beam with OAM, we can modify one of the plane waves by imposing a spiral phase $e^{il\phi}$ ¹⁶, which produces an interference known as a forked diffraction grating; in the right side of Figure 5, the first diffracted order from an incident Gaussian beam will be a Hypergeometric-Gaussian (HyGG) mode₂ with the programmed l -value. Figure 6 shows example data of a transverse intensity profile of an HyGG beam with $l = 4$ generated by a similar forked grating.

The device used to project these holograms is known as a spatial light modulator (SLM). In the case of the presented data, we have used a rewired classroom projector. Conceptually, once the laser beam is incident on the grating displayed on the SLM, the Gaussian beam “undoes” one of the plane waves in the interference pattern, and then the first diffracted order is left with just the desired signal beam. The diffraction grating transmits the original beam at the 0th diffracted order, and symmetrically diffracts the signal beam in the higher-order diffracted modes. The SLM in this case performs via transmission. A diagram of our transmission SLM procedure is shown in Figure 7.

The power in generating structured light with holograms displayed on SLMs is that we can program any mode we desire, limited only by capabilities of programming those modes analytically or numerically. This has been crucial for us in our study of the topological fluid of light (TFL), as we can program and generate any arbitrary array of vortices, as seen in Figure 8. These techniques of generation and measurement¹⁴ have given us a unique toolkit to dive into this new area of research. For instance, we wish to study vortices as quasiparticles under a quantum field theory—like formalism—in particle physics, it is well known that oppositely charged

particles and antiparticles will attract and then annihilate with each other upon impact. This work is still new and fresh, but it carries the momentum of the concepts and techniques developed thus far, as partly presented in this article.

3 ACKNOWLEDGEMENTS

Dr. Mark Siemens, Jasmine Andersen, Sam Alperin, Abi Grotelueschen, William Holtzmann, and Leah Huzjak have all contributed to the work presented in this article. I have cited their work directly in the article, either for data, figure-making, or publications.

4 EDITOR’S NOTES

This work was adapted from a senior thesis and has been condensed for publication. Contact DUURJ staff for the full publication.

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